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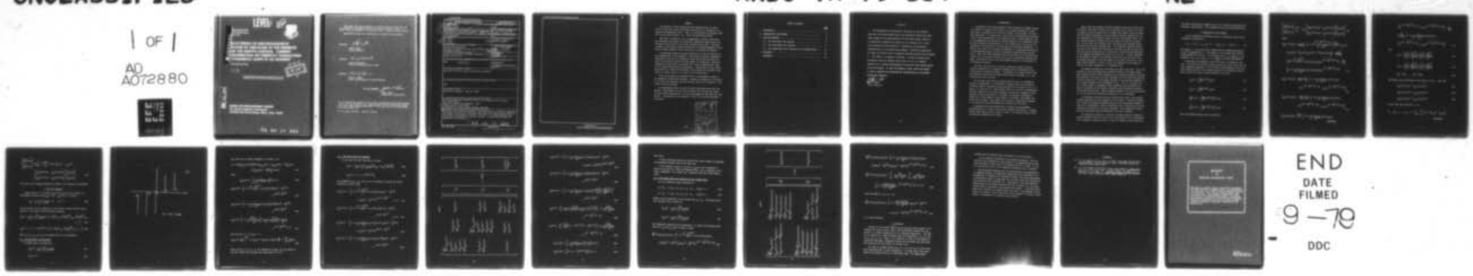

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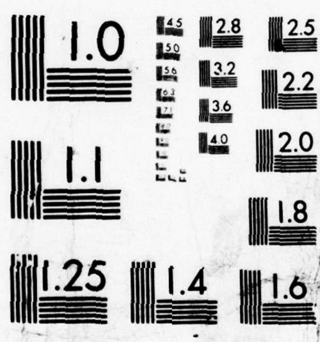

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Final Technical Report
June 1979

SCATTERING OF ELECTROMAGNETIC WAVES BY OBSTACLES IN THE PRESENCE OF THE EARTH'S SURFACE: CURRENT DISTRIBUTION ON PERFECTLY CONDUCTING CYLINDRICAL LOOPS IN AN INCIDENT

Harvard University

T. T. Wu
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Air Force Systems Command
Griffiss Air Force Base, New York 13441

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The current distribution excited by an external field on a perfectly conducting cylindrical loop with zero thickness placed above the earth's surface with its axis normal to the earth is obtained. The case of two coupled coaxial loops at different separations has also been solved. A variational principle is employed in the derivation.

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SUMMARY

The solution to this problem has been separated into two parts. First, we find the current distribution excited on the scatterer by an external electromagnetic wave. Then, we can calculate the radiated field due to the excited current, i.e., solve the Sommerfeld problem.

The earth is idealized to be a homogeneous, isotropic half-space. The scatterers are chosen to be perfectly conducting coaxial loops with arbitrary cross section in the plane containing their axis. This axis is normal to the earth's surface and there is rotational symmetry so that Fourier series expansion can be carried out to simplify the problem.

A variational approach has been formulated to give the surface current distribution on the scatterer. The particular cases of scatterers consisting of one and two cylindrical loops (loops that are short sections of cylindrical tubes) have been worked out explicitly. Numerical computations still have to be carried out to see if higher-order terms of the current distribution are necessary. Calculations for the current distributions of loops with different cross sections are desirable especially for planar loops, i.e., loops cut from plane discs.

Prior to obtaining the scattered field, a comprehensive review of the Sommerfeld problem was initiated. Despite an abundance of literature in the area, the available solution does not seem to be conclusive. This review is now in progress.

An experimental setup to check the newly developed theory has been devised. Measurements of the field scattered from a single loop above the earth when irradiated by a 1.5 GHz electromagnetic wave have been completed. Preparations are in progress to carry out the same experiment over water.

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EVALUATION

1. The experimental and theoretical work done on this contract shows how the backscattered energy from horizontal dipoles and small loops changes as the characteristics of the earth change. (R2B,7.1) An experimental program was carried out at a frequency of 1.5 GHz. The scattered field was measured in elevation as the electrical characteristics of the ground were changed. Theoretical expressions for loops and dipoles near an imperfect groundplane were also developed.

2. This work is in support of TPO R2B. In order to extract more information from the returned radar signal as well as improved detectability a better understanding of the return from simple objects must be developed. As more low frequency radars (where the target is nearly resonant) are developed the characteristics of the earth become more important.

Otho E. Kerr

OTHO E. KERR
Project Engineer

1. INTRODUCTION

A knowledge of the electromagnetic field scattered by targets located on or above the surface of the earth is of primary importance in radar. This field is radiated by the currents induced in the target by the incident plane wave. It follows that a first step in evaluating the scattered field is the determination of the induced currents. These depend not only on the geometrical and electrical characteristics of the scattering target and the direction and polarization of the incident wave, but also on the proximity of the earth with its conducting and dielectric properties. In effect, the current in the target corresponds to a continuous distribution of infinitesimal dipoles, each with appropriate amplitude and relative phase. Thus, the total scattered field is the vector sum of the contributions by all of the small dipoles, each with its particular amplitude and phase. The field of one of the infinitesimal dipoles is the solution of the Sommerfeld problem.

The thin circular loop is the most convenient target to analyze initially because of its rotational symmetry and its usefulness in the construction of composite targets. Furthermore, the current distribution and scattered field are well known when the loop is isolated, i.e., sufficiently far from the earth.

The circular loop is also a simple target for use in a systematic program of measurement. As described in greater detail in the Interim Scientific Report, "Scattering from Obstacles over the Earth," by R. W. P. King, H. M. Lee, and L. C. Shen (August 1978) [1], one set of measurements has been completed of the field scattered by horizontal resonant and non-resonant circular loops and by a straight wire one-half wavelength in length over a wide range of sizes, angles of incidence, and heights outdoors over the earth. That report also contains a theoretical treatment of the scattering from horizontal-wire antennas over the earth.

An apparatus has been constructed to continue the measurements systematically over a tank of water with variable conductivity. A comprehensive experimental study is about to begin. The results should provide an essential comparison with theory.

Among loops that possess rotational symmetry around the z-axis, which is normal to the earth' surface, the cylindrical loop, which is cut from a cylinder of zero thickness with its height much smaller than its radius and the incident wavelength, has the simplest integral equations for the two components of the surface current excited by an external field. The two coupled integral equations can be decoupled, but this is of little use. We are faced with constants that can assume very different values and affect the solution qualitatively. The dielectric constant and conductivity of the earth vary from those of dry sand to those of sea water. The loop can be located close to the surface or raised through multiples of the incident wavelength until high up in the air. A series expansion of the kernel in terms of the smallest scale factor for the loop, viz., its height multiplied by the wave number in the earth, may not be a very small number compared to unity. Hence, the expansion of the kernel in terms of its small parameters is a forbidding task.

The Maxwell equations for free space are simple and clear. But in the presence of even the geometrically simplest objects, they seldom permit exact general solutions when subjected to the requisite boundary conditions. When applied to the loop, the axial dependence of the current distribution should be simple due to the assumed smallness of the vertical dimension of the cylinder compared to the wavelength in air. That is, we expect that only a small number of parameters whose magnitudes are governed by the Maxwell equations and the boundary conditions should adequately describe the current distribution. These parameters depend on the size and location of the loop and on the electrical properties of its environment.

A variational principle has been obtained that leads to the coupled integral equations for the components of the current. Instead of attempting to solve the integral equations directly, the variational integral is used with an approximate current distribution that is described by the few parameters we recognize as the most important. This changes the problem into the relatively simple determination of several constants.

The variational approach is powerful not only because it brings the several constants under our control, but also because it can be systematically extended to multiple loops, to loops of different cross sections by the plane containing the symmetry axis, and to multiple loops each with arbitrary cross sections. This extension has already been carried out.

2. FORMULATION OF THE PROBLEM

In the cylindrical coordinates (ρ,ϕ,z) a cylindrical loop of zero thickness is described by:

$$\rho = b, \quad 0 \le h - w \le z \le h + w \quad , \quad |k_1 w| << 1 \quad , \quad w/b << 1 \tag{1}$$

The region $z > 0$ is filled with a medium with the complex permittivity $\tilde{\varepsilon}_1$ and the permeability μ_1. The region $z < 0$ is filled with a medium characterized by $\tilde{\varepsilon}_2$ and μ_2. It is assumed that $\tilde{\varepsilon}_1$, μ_1, and $\tilde{\varepsilon}_2$, μ_2 are constants. ($\tilde{\varepsilon}_j = \varepsilon_j + i\sigma_j/\omega$, $j = 1, 2$, where σ_j is the conductivity.)

An external electromagnetic wave $\vec{E}^{(i)}(\rho,\phi,z)$ is incident on the loop. It excites the surface currents $K_\phi(\phi,z)$ and $K_z(\phi,z)$ such that the total tangential component of the electric field vanishes on the loop. This condition leads to the integral equation for the surface current. First, define the Fourier series expansions as follows:

$$K_{\phi_n}(z) \equiv \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} e^{-in\phi} K_\phi(\phi,z) \tag{2}$$

$$K_{z_n}(z) \equiv \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} e^{-in\phi} K_z(\phi,z) \tag{3}$$

$$E^{(i)}_{\phi_n}(\rho,z) \equiv \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} e^{-in\phi} E^{(i)}_\phi(\rho,\phi,z) \tag{4}$$

$$E^{(i)}_{z_n}(\rho,z) \equiv \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} e^{-in\phi} E^{(i)}_z(\rho,\phi,z) \tag{5}$$

Then the integral equation can be written as:

$$
\begin{bmatrix} -E^{(i)}_{\phi_n}(b,z) \\ -E^{(i)}_{z_n}(b,z) \end{bmatrix} = \int_{h-w}^{h+w} b\, dz_0 \begin{bmatrix} G_{\phi\phi_n}(b,z;b,z_0) & G_{\phi z_n}(b,z;b,z_0) \\ G_{z\phi_n}(b,z;b,z_0) & G_{zz_n}(b,z;b,z_0) \end{bmatrix} \begin{bmatrix} K_{\phi_n}(z_0) \\ K_{z_n}(z_0) \end{bmatrix} \tag{6}
$$

where

$$
\begin{aligned}
G_{\phi\phi_n}(\rho,z;\rho_0,z_0) = {} & \frac{i}{2\omega\bar{\varepsilon}_1\rho\rho_0} \int_0^\infty \lambda\, d\lambda \frac{e^{-(\lambda^2-k_1^2)^{1/2}|z-z_0|}}{(\lambda^2-k_1^2)^{1/2}} \left\{ \frac{k_1^2\rho\rho_0}{2} \left[J_{n-1}(\lambda\rho)\right.\right. \\
& \times \left.\left. J_{n-1}(\lambda\rho_0) + J_{n+1}(\lambda\rho)J_{n+1}(\lambda\rho_0)\right] - n^2 J_n(\lambda\rho)J_n(\lambda\rho_0) \right\} + \frac{in^2}{2\omega\bar{\varepsilon}_1\rho\rho_0} \\
& \times \int_0^\infty \lambda d\lambda \frac{(\lambda^2-k_1^2)^{1/2}}{\lambda} R_\varepsilon(\lambda) J_n(\lambda\rho) J_n(\lambda\rho_0) e^{-(\lambda^2-k_1^2)^{1/2}(z+z_0)} + \frac{i\omega\mu_1}{2} \\
& \times \int_0^\infty \lambda\, d\lambda \frac{1}{(\lambda^2-k_1^2)^{1/2}} R_\mu(\lambda) J'_n(\lambda\rho) J'_n(\lambda\rho_0) e^{-(\lambda^2-k_1^2)^{1/2}(z+z_0)}
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
G_{\phi z_n}(\rho,z;\rho_0,z_0) = {} & \frac{1}{2\omega\bar{\varepsilon}_1} \frac{n}{\rho} \frac{\partial}{\partial z_0} \int_0^\infty \lambda\, d\lambda \frac{1}{(\lambda^2-k_1^2)^{1/2}} J_n(\lambda\rho) J_n(\lambda\rho_0) \\
& \times \left[e^{-(\lambda^2-k_1^2)^{1/2}|z-z_0|} - R_\varepsilon(\lambda) e^{-(\lambda^2-k_1^2)^{1/2}(z+z_0)} \right]
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
G_{z\phi_n}(\rho,z;\rho_0,z_0) = {} & -\frac{1}{2\omega\bar{\varepsilon}_1} \frac{n}{\rho_0} \frac{\partial}{\partial z} \int_0^\infty \lambda\, d\lambda \frac{1}{(\lambda^2-k_1^2)^{1/2}} J_n(\lambda\rho) J_n(\lambda\rho_0) \\
& \times \left[e^{-(\lambda^2-k_1^2)^{1/2}|z-z_0|} - R_\varepsilon(\lambda) e^{-(\lambda^2-k_1^2)^{1/2}(z+z_0)} \right]
\end{aligned} \tag{9}
$$

$$
G_{zz_n}(\rho,z;\rho_0,z_0) = \frac{i\omega\mu_1}{2} \int_0^\infty \lambda\, d\lambda \frac{1}{(\lambda^2-k_1^2)^{1/2}} J_n(\lambda\rho) J_n(\lambda\rho_0)
$$

[CONTINUED]

$$\times \left[e^{-(\lambda^2 - k_1^2)^{1/2}|z - z_0|} + R_\varepsilon(\lambda) e^{-(\lambda^2 - k_1^2)^{1/2}(z + z_0)} \right] - \frac{1}{2\omega\tilde{\varepsilon}_1}$$

$$\times \frac{\partial^2}{\partial z\, \partial z_0} \int_0^\infty \lambda\, d\lambda \frac{1}{(\lambda^2 - k_1^2)^{1/2}} J_n(\lambda\rho) J_n(\lambda\rho_0)$$

$$\times \left[e^{-(\lambda^2 - k_1^2)^{1/2}|z - z_0|} - R_\varepsilon(\lambda) e^{-(\lambda^2 - k_1^2)^{1/2}(z + z_0)} \right] \qquad (10)$$

with

$$R_\varepsilon(\lambda) \equiv \frac{\tilde{\varepsilon}_2(\lambda^2 - k_1^2)^{1/2} - \tilde{\varepsilon}_1(\lambda^2 - k_2^2)^{1/2}}{\tilde{\varepsilon}_2(\lambda^2 - k_1^2)^{1/2} + \tilde{\varepsilon}_1(\lambda^2 - k_2^2)^{1/2}} \qquad (11)$$

$$R_\mu(\lambda) \equiv \frac{\mu_2(\lambda^2 - k_1^2)^{1/2} - \mu_1(\lambda^2 - k_2^2)^{1/2}}{\mu_2(\lambda^2 - k_1^2)^{1/2} + \mu_1(\lambda^2 - k_2^2)^{1/2}} \qquad (12)$$

$$k_1^2 = \omega^2 \mu_1 \tilde{\varepsilon}_1 \quad , \quad k_2^2 = \omega^2 \mu_2 \tilde{\varepsilon}_2 \qquad (13)$$

The branch cuts and integration path appear in Fig. 1. Note that

$$G_{\phi\phi_n}(\rho,z;\rho_0,z_0) = G_{\phi\phi_{-n}}(\rho_0,z_0;\rho,z) \qquad (14)$$

$$G_{\phi z_n}(\rho,z;\rho_0,z_0) = G_{z\phi_{-n}}(\rho_0,z_0;\rho,z) \qquad (15)$$

$$G_{zz_n}(\rho,z;\rho_0,z_0) = G_{zz_{-n}}(\rho_0,z_0;\rho,z) \qquad (16)$$

We know that the functional V, viz.,

$$V(\ldots K_{\phi_n} \ldots, \ldots K_{z_n} \ldots) = \sum_{n=-\infty}^{\infty} \int_{h-w}^{h+w} b\, dz\, [K_{\phi_{-n}}(z) \quad K_{z_{-n}}(z)]$$

[CONTINUED]

$$\times \begin{bmatrix} E^{(i)}_{\phi_n}(b,z) \\ E^{(i)}_{z_n}(b,z) \end{bmatrix} + \frac{1}{2} \sum_{n=-\infty}^{\infty} b^2 \iint_{h-w}^{h+w} dz\, dz_0\, [K_{\phi_{-n}}(z) \quad K_{z_{-n}}(z)]$$

$$\times \begin{bmatrix} G_{\phi\phi_n}(b,z;b,z_0) & G_{\phi z_n}(b,z;b,z_0) \\ G_{z\phi_n}(b,z;b,z_0) & G_{zz_n}(b,z;b,z_0) \end{bmatrix} \begin{bmatrix} K_{\phi_n}(z_0) \\ K_{z_n}(z_0) \end{bmatrix} \tag{17}$$

will give us the integral equations by means of the calculus of variations.

3. THE TEST FUNCTION

Because $|k_1 w| << 1$, we expect $K_{\phi_n}(z)$ and $K_{z_n}(z)$ to vary slowly in z. Furthermore, we expect that $K_{z_n}(h - w) = K_{z_n}(h + w) = 0$, while

$$K_{\phi_n} \to \frac{1}{[w \mp (z-h)]^{1/2}} \quad \text{as} \quad z \to h \pm w \tag{18}$$

We also expect that the components of the surface current are well represented by a few terms of the following functions:

$$K_{\phi_n}(z) = \frac{I_n}{\pi[w^2 - (z-h)^2]^{1/2}} \left[1 + \frac{z-h}{w} s_{n_1} + \left(\frac{z-h}{w}\right)^2 s_{n_2} + \left(\frac{z-h}{w}\right)^3 s_{n3} + \ldots\right] \tag{19}$$

$$K_{z_n}(z) = [1 - (z-h)^2/w^2]^{1/2} K_n \left[1 + \frac{z-h}{w} t_{n_1} + \left(\frac{z-h}{w}\right)^2 t_{n_2} + \ldots\right] \tag{20}$$

where I_n, K_n, s_{n_i}, and t_{n_i} are parameters yet to be determined.

3.1. The zero-order test function.

To lowest order, we assume that

$$K_{\phi_n}(z) = \frac{I_n}{\pi[w^2 - (z-h)^2]^{1/2}} \tag{21}$$

$$K_{z_n}(z) = 0 \tag{22}$$

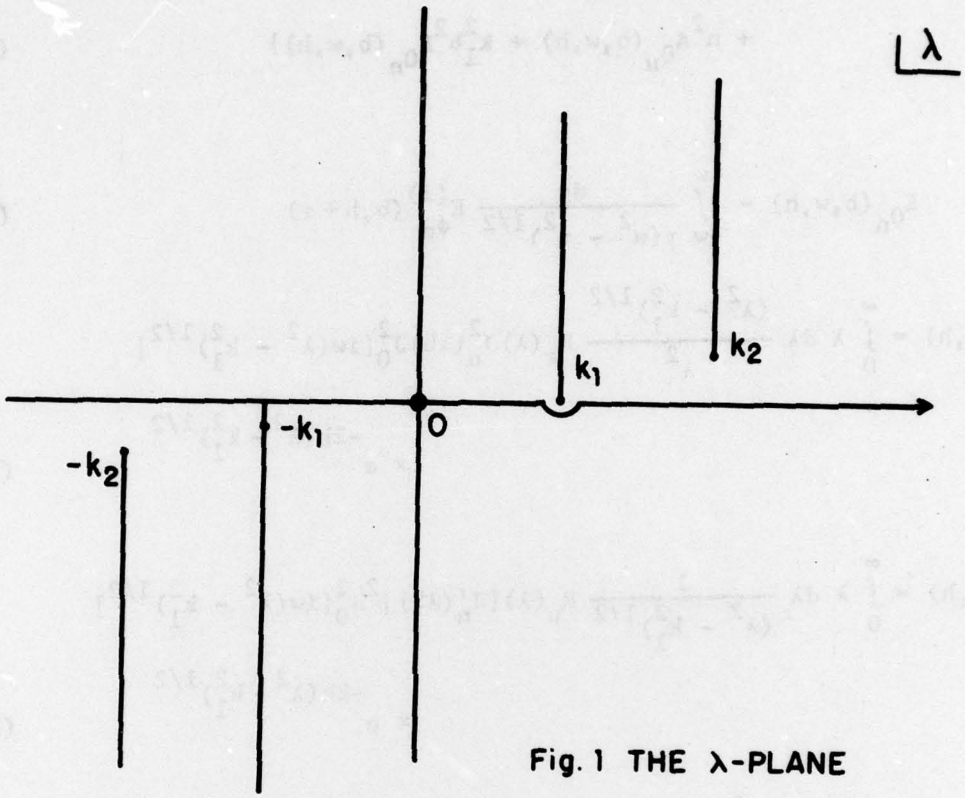

Fig. 1 THE λ-PLANE

With this set of current components, we obtain I_n as:

$$I_n = 2i\omega\tilde{\varepsilon}_1 bE_{0_n}(b,w,h)\{(k_1^2b^2/2)[F_{0_{n-1}}(b,w) + F_{0_{n+1}}(b,w)] - n^2F_{0_n}(b,w) + n^2A_{0_n}(b,w,h) + k_1^2b^2B_{0_n}(b,w,h)\} \tag{23}$$

where

$$E_{0_n}(b,w,h) = \int_{-w}^{w} \frac{d\tau}{\pi(w^2 - \tau^2)^{1/2}} E_{\phi_n}^{(i)}(b,h+\tau) \tag{24}$$

$$A_{0_n}(b,w,h) = \int_0^\infty \lambda\, d\lambda \frac{(\lambda^2 - k_1^2)^{1/2}}{\lambda^2} R_\varepsilon(\lambda) J_n^2(\lambda b) J_0^2[iw(\lambda^2 - k_1^2)^{1/2}] \times e^{-2h(\lambda^2 - k_1^2)^{1/2}} \tag{25}$$

$$B_{0_n}(b,w,h) = \int_0^\infty \lambda\, d\lambda \frac{1}{(\lambda^2 - k_1^2)^{1/2}} R_\mu(\lambda)[J_n'(\lambda b)]^2 J_0^2[iw(\lambda^2 - k_1^2)^{1/2}] \times e^{-2h(\lambda^2 - k_1^2)^{1/2}} \tag{26}$$

$$F_{0_n}(b,w) = \int_0^\infty \lambda\, d\lambda \frac{J_n^2(\lambda b)}{(\lambda^2 - k_1^2)^{1/2}} \iint_{-w}^{w} \frac{d\tau\, d\tau_0}{\pi^2(w^2 - \tau^2)^{1/2}(w^2 - \tau_0^2)^{1/2}} \times e^{-(\lambda^2 - k_1^2)^{1/2}|\tau - \tau_0|} \tag{27}$$

Note that for $w \ll b$, $|k_1 w| \ll 1$,

$$F_{0_n}(b,w) \simeq \frac{i}{2b} \int_0^{2k_1 b} dx\, [J_{2n}(x) + i\Omega_{2n}(x)] + \frac{1}{\pi b}\left[\ln \frac{16b}{w} - 2 \sum_{m=1}^{|n|} \frac{1}{2m - 1}\right] \tag{28}$$

Hence, when $\tilde{\varepsilon}_1 = \tilde{\varepsilon}_2$, $\mu_1 = \mu_2$, we reproduce the result for the theory of the thin tubular loop antenna with width equal to 4w [2].

3.2. The first-order test function.

We can write the test functions as follows:

$$K_{\phi_n}(z) = \frac{1}{\pi[w^2 - (z - h)^2]^{1/2}} \left[I_{10_n} + I_{11_n} \left(\frac{z - h}{w} \right) \right] \tag{29}$$

$$K_{z_n}(z) = [1 - (z - h)^2/w^2]^{1/2} K_n \tag{30}$$

The parameters I_{10_n}, I_{11_n}, and K_n can be obtained by solving the linear equations in Table 1 with

$$A_{1_n}(b,w,h) = i \int_0^{\infty} \lambda \, d\lambda \frac{(\lambda^2 - k_1^2)^{1/2}}{\lambda^2} R_{\varepsilon}(\lambda) J_n^2(\lambda b) J_0[iw(\lambda^2 - k_1^2)^{1/2}]$$

$$\times J_1[iw(\lambda^2 - k_1^2)^{1/2}] e^{-2h(\lambda^2 - k_1^2)^{1/2}} \quad , \quad n \neq 0 \tag{31}$$

$$A_{2_n}(b,w,h) = - \int_0^{\infty} \lambda \, d\lambda \frac{(\lambda^2 - k_1^2)^{1/2}}{\lambda^2} R_{\varepsilon}(\lambda) J_n^2(\lambda b) J_1^2[iw(\lambda^2 - k_1^2)^{1/2}]$$

$$\times e^{-2h(\lambda^2 - k_1^2)^{1/2}} \quad , \quad n \neq 0 \tag{32}$$

$$B_{1_n}(b,w,h) = i \int_0^{\infty} \lambda \, d\lambda \frac{1}{(\lambda^2 - k_1^2)^{1/2}} R_{\mu}(\lambda) [J_n'(\lambda b)]^2 J_0[iw(\lambda^2 - k_1^2)^{1/2}]$$

$$\times J_1[iw(\lambda^2 - k_1^2)^{1/2}] e^{-2h(\lambda^2 - k_1^2)^{1/2}} \tag{33}$$

$$B_{2_n}(b,w,h) = - \int_0^{\infty} \lambda \, d\lambda \frac{1}{(\lambda^2 - k_1^2)^{1/2}} R_{\mu}(\lambda) [J_n'(\lambda b)]^2 J_1^2[iw(\lambda^2 - k_1^2)^{1/2}]$$

$$\times e^{-2h(\lambda^2 - k_1^2)^{1/2}} \tag{34}$$

TABLE 1

$$\begin{bmatrix} \begin{array}{l} \frac{k_1^2b^2}{2}[F_{0_{n-1}}(b,w) + \\ F_{0_{n+1}}(b,w)] - \\ n^2F_{0_n}(b,w) + \\ n^2A_{0_n}(b,w,h) + \\ k_1^2b^2B_{0_n}(b,w,h) \end{array} & \begin{array}{l} n^2A_{1_n}(b,w,h) + \\ k_1^2b^2B_{1_n}(b,w,h) \end{array} & inC_{1_n}(b,w,h) \\ \begin{array}{l} n^2A_{1_n}(b,w,h) + \\ k_1^2b^2B_{1_n}(b,w,h) \end{array} & \begin{array}{l} \frac{k_1^2b^2}{2}[F_{2_{n-1}}(b,w) + \\ F_{2_{n+1}}(b,w)] - \\ n^2F_{2_n}(b,w) + \\ n^2A_{2_n}(b,w,h) + \\ k_1^2b^2B_{2_n}(b,w,h) \end{array} & \begin{array}{l} -in[F_{2_n}(b,w) - \\ C_{2_n}(b,w,h)] \end{array} \\ -inC_{1_n}(b,w,h) & \begin{array}{l} in[F_{2_n}(b,w) - \\ C_{2_n}(b,w,h)] \end{array} & \begin{array}{l} k_1^2w^2G_{0_n}(b,w) - \\ F_{2_n}(b,w) + \\ k_1^2w^2D_{0_n}(b,w,h) + \\ C_{2_n}(b,w,h) \end{array} \end{bmatrix} \begin{bmatrix} I_{10_n} \\ I_{11_n} \\ \pi bK_n \end{bmatrix} = 2i\omega\tilde{\epsilon}_1 b \begin{bmatrix} E_{0_n}(b,w,h) \\ E_{1_n}(b,w,h) \\ \frac{V_{0_n}(b,w,h)}{\pi b} \end{bmatrix}$$

$$C_{1_n}(b,w,h) = i \int_0^\infty \lambda\, d\lambda \frac{1}{(\lambda^2 - k_1^2)^{1/2}} R_\varepsilon(\lambda) J_n^2(\lambda b) J_0[iw(\lambda^2 - k_1^2)^{1/2}]$$

$$\times J_1[iw(\lambda^2 - k_1^2)^{1/2}] e^{-2h(\lambda^2 - k_1^2)^{1/2}} \tag{35}$$

$$C_{2_n}(b,w,h) = - \int_0^\infty \lambda\, d\lambda \frac{1}{(\lambda^2 - k_1^2)^{1/2}} R_\varepsilon(\lambda) J_n^2(\lambda b) J_1^2[iw(\lambda^2 - k_1^2)^{1/2}]$$

$$\times e^{-2h(\lambda^2 - k_1^2)^{1/2}} \tag{36}$$

$$D_{0_n}(b,w,h) = - \int_0^\infty \lambda\, d\lambda \frac{1}{(\lambda^2 - k_1^2)^{1/2}} R_\varepsilon(\lambda) J_n^2(\lambda b) \left[\frac{J_1[iw(\lambda^2 - k_1^2)^{1/2}]}{w(\lambda^2 - k_1^2)^{1/2}} \right]^2$$

$$\times e^{-2h(\lambda^2 - k_1^2)^{1/2}} \tag{37}$$

$$F_{2_n}(b,w) = \iint_{-w}^{w} \frac{d\tau\, d\tau_0}{\pi^2 (w^2 - \tau^2)^{1/2} (w^2 - \tau_0^2)^{1/2}} \left(\frac{\tau}{w}\right)\left(\frac{\tau_0}{w}\right) \int_0^\infty \lambda\, d\lambda \frac{J_n^2(\lambda b)}{(\lambda^2 - k_1^2)^{1/2}}$$

$$\times e^{-(\lambda^2 - k_1^2)^{1/2} |\tau - \tau_0|} \tag{38}$$

$$G_{0_n}(b,w) = \iint_{-w}^{w} \frac{d\tau\, d\tau_0}{\pi^2 w^2} [1 - \tau^2/w^2]^{1/2} [1 - \tau_0^2/w^2]^{1/2} \int_0^\infty \lambda\, d\lambda \frac{J_n^2(\lambda b)}{(\lambda^2 - k_1^2)^{1/2}}$$

$$\times e^{-(\lambda^2 - k_1^2)^{1/2} |\tau - \tau_0|} \tag{39}$$

$$E_{1_n}(b,w,h) = \int_{-w}^{w} \frac{d\tau}{\pi(w^2 - \tau^2)^{1/2}} \left(\frac{\tau}{w}\right) E_{\phi_n}^{(i)}(b, h+\tau) \tag{40}$$

$$V_{0_n}(b,w,h) = \int_{-w}^{w} d\tau\, [1 - \tau^2/w^2]^{1/2} E_{z_n}^{(i)}(b, h+\tau) \tag{41}$$

Note that:

1) This variational principle provides the correct weight in averaging the externally applied field on the loop; and

2) The integrals $F_{0_n}(b,w)$, $F_{2_n}(b,w)$, $G_{0_n}(b,w)$ can be expanded in powers of $k_1 w$ and w/b. After the expansion the τ and τ_0 integrals can be easily integrated. This leaves the λ integral which can be done numerically.

3.3. The lowest-order test function for two coupled loops.

For two cylindrical loops described by:

$$\rho_1 = b_1 \ , \quad 0 \leq h_1 - w_1 \leq z_1 \leq h_1 + w_1 \ , \quad |k_1 w_1| << 1 \tag{42}$$

$$\rho_2 = b_2 \ , \quad 0 \leq h_2 - w_2 \leq z_2 \leq h_2 + w_2 \ , \quad |k_1 w_2| << 1 \tag{43}$$

Without loss of generality, we may assume that $h_1 \geq h_2$. The lowest-order test functions can be chosen as:

$$K_{1\phi_n}(z) = \frac{I_{1_n}}{\pi[w^2 - (z-h)^2]^{1/2}} \tag{44}$$

$$K_{2\phi_n}(z) = \frac{I_{2_n}}{\pi[w^2 - (z-h)^2]^{1/2}} \tag{45}$$

The variational integral can be constructed. It leads to the linear equations for I_{1_n} and I_{2_n} given in Table 2, where

$$A_{0_n}^{(2)}(b_1,b_2;w_1,w_2;h_1,h_2) = \int_0^\infty \lambda \, d\lambda \, \frac{(\lambda^2 - k_1^2)^{1/2}}{\lambda^2} R_\epsilon(\lambda) J_n(\lambda b_1) J_n(\lambda b_2)$$
$$\times J_0[iw_1(\lambda^2 - k_1^2)^{1/2}] J_0[iw_2(\lambda^2 - k_1^2)^{1/2}] e^{-(\lambda^2 - k_1^2)^{1/2}(h_1 + h_2)} \tag{46}$$

TABLE 2

$$\left[\begin{array}{ll}
\begin{array}{l}\dfrac{k_1^2 b_1^2}{2}\,[F_{0_{n-1}}(b_1,w_1) + \\ F_{0_{n+1}}(b_1,w_1)] - n^2 F_{0_n}(b_1,w_1) + \\ n^2 A_{0_n}(b_1,w_1,h_1) + k_1^2 b_1^2 B_{0_n}(b_1,w_1,h_1)\end{array} & \begin{array}{l}\dfrac{k_1^2 b_1 b_2}{2}\,[F^{(2)}_{0_{n-1}}(b_1,b_2;w_1,w_2;h_1,h_2) \\ + F^{(2)}_{0_{n+1}}(b_1,b_2;w_1,w_2;h_1,h_2)] - \\ n^2 F^{(2)}_{0_n}(b_1,b_2;w_1,w_2;h_1,h_2) + \\ n^2 A^{(2)}_{0_n}(b_1,b_2;w_1,w_2;h_1,h_2) + \\ k_1^2 b_1 b_2 B^{(2)}_{0_n}(b_1,b_2;w_1,w_2;h_1,h_2)\end{array} \\
 & \\
\begin{array}{l}\dfrac{k_1^2 b_1 b_2}{2}\,[F^{(2)}_{0_{n-1}}(b_1,b_2;w_1,w_2;h_1,h_2) \\ + F^{(2)}_{0_{n+1}}(b_1,b_2;w_1,w_2;h_1,h_2)] - \\ n^2 F^{(2)}_{0_n}(b_1,b_2;w_1,w_2;h_1,h_2) + \\ n^2 A^{(2)}_{0_n}(b_1,b_2;w_1,w_2;h_1,h_2) + \\ k_1^2 b_1 b_2 B^{(2)}_{0_n}(b_1,b_2;w_1,w_2;h_1,h_2)\end{array} & \begin{array}{l}\dfrac{k_1^2 b_2^2}{2}\,[F_{0_{n-1}}(b_2,w_2) + \\ F_{0_{n+1}}(b_2,w_2)] - n^2 F_{0_n}(b_2,w_2) + \\ n^2 A_{0_n}(b_2,w_2,h_2) + k_1^2 b_2^2 B_{0_n}(b_2,w_2,h_2)\end{array}
\end{array}\right]
\left[\begin{array}{c} I^{(2)}_{1_n} \\ \\ I^{(2)}_{2_n} \end{array}\right]
= 2i\omega\bar{\varepsilon}_1
\left[\begin{array}{c} b_1 E_{0_n}(b_1,w_1,h_1) \\ \\ b_2 E_{0_n}(b_2,w_2,h_2) \end{array}\right]$$

$$B_{0_n}^{(2)}(b_1,b_2;w_1,w_2;h_1,h_2) = \int_0^\infty \lambda\, d\lambda \frac{1}{(\lambda^2 - k_1^2)^{1/2}} R_\mu(\lambda) J_n'(\lambda b_1) J_n'(\lambda b_2)$$

$$\times J_0[iw_1(\lambda^2 - k_1^2)^{1/2}] J_0[iw_2(\lambda^2 - k_1^2)^{1/2}] e^{-(\lambda^2 - k_1^2)^{1/2}(h_1 + h_2)} \qquad (47)$$

$$F_{0_n}^{(2)}(b_1,b_2;w_1,w_2;h_1,h_2) = \int_{-w_1}^{w_1} \frac{d\tau_1}{\pi(w_1^2 - \tau_1^2)^{1/2}} \int_{-w_2}^{w_2} \frac{d\tau_2}{\pi(w_2^2 - \tau_2^2)^{1/2}}$$

$$\times \int_0^\infty \lambda\, d\lambda \frac{J_n(\lambda b_1) J_n(\lambda b_2)}{(\lambda^2 - k_1^2)^{1/2}} e^{-(\lambda^2 - k_1^2)^{1/2} |h_1 - h_2 + \tau_1 - \tau_2|} \qquad (48)$$

Note that when $h_1 - h_2 \geq w_1 + w_2$,

$$F_{0_n}^{(2)}(b_1,b_2;w_1,w_2;h_1,h_2) = \int_0^\infty \lambda\, d\lambda \frac{J_n(\lambda b_1) J_n(\lambda b_2)}{(\lambda^2 - k_1^2)^{1/2}} J_0[iw_1(\lambda^2 - k_1^2)^{1/2}]$$

$$\times J_0[iw_2(\lambda^2 - k_1^2)^{1/2}] e^{-(\lambda^2 - k_1^2)^{1/2}(h_1 - h_2)} \qquad (49)$$

is a single integral.

4. DISCUSSION

Except for the constant $F_{0_n}^{(2)}(b_1,b_2;w_1,w_2;h_1,h_2)$ which appears in the solution for coupled loops, each of the coefficients involved has been reduced to a single integral and can be evaluated numerically. Even $F_{0_n}^{(2)}$ can be reduced to a single integral under the situations $h_1 - h_2 \geq w_1 + w_2$ or $h_1 = h_2$, $w_1 = w_2$. Hence, it is desirable to carry out the numerical integration and make a comparison between the zero-order and first-order test functions for the current on a single loop.

The case of a single planar loop, i.e., one cut out of a zero thickness conducting disc, is very interesting. To lowest order, the result should be the same as that of a cylindrical loop. For higher-order currents and for coupled loops, the answer is far from obvious.

We are primarily interested in the scattering problem. Now that there are test functions for the currents it is essential that we look into the effect of each term of the function on the far field. A careful and detailed review of the Sommerfeld problem is now in progress.

The variational approach can be justified either self-consistently by comparing results of successively higher-order test functions or experimentally by comparing the calculated results with direct measurements. An experimental setup for measuring the scattered fields from cylindrical brass loops has been completed [1]. Unfortunately, the C203 generator, manufactured by the Applied Microwave Division of EPSCO, Inc., has never worked stably or delivered a high enough power output. It has been determined that, although the Siemens YD1381 tube used in the generator may be aging, the resonant cavity itself must first be rebuilt to correct a faulty design. The generator is currently being repaired.

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